Solution by A. M. Harding, University of Arkansas.

A solution of this problem will be found in Jean's *Theoretical Mechanics*, pages 80–85. It is there shown that the length of the wire is given by

$$s = \frac{c}{2}\left(e^{x/c} - e^{-x/c}\right).$$

If we expand $e^{x/c}$ and $e^{-x/c}$ as power series in x and neglect all powers of x higher than the third we obtain the desired result.

Similarly solved by Walter C. Eells and R. M. Mathews.

302. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A ball is projected from a given point at a given inclination β towards a vertical wall; determine the velocity so that after striking the wall the ball may return to the point of projection.

Solution by Paul Capron, Annapolis, Maryland.

Let v = the required velocity, a = the distance of the point of projection from the wall. After t seconds, the ball will have traversed x horizontally, y vertically, where

$$x = v\cos\beta\cdot t,\quad y = v\sin\beta\cdot t - \tfrac{1}{2}gt^2.$$

Its path will make the angle α with the horizontal, where $\tan\alpha = \tan\beta - (gt/v\cos\beta)$, and its component velocities will be $v\cos\beta$ horizontally, $v\sin\beta - gt$ vertically. Subscripting the values at the instant of impact, we have

$$x_1 = a,\quad t_1 = (a/v\cos\beta),\quad y_1 = a\tan\beta - (a^2g/2v^2)\sec^2\beta,\quad \tan\alpha_1 = \tan\beta - (ag/v^2)\sec^2\beta,$$
$$v_1^2 = v^2 - 2ag\tan\beta + (a^2g^2/v^2)\sec^2\beta = v^2 - ag\,(\tan\alpha_1 + \tan\beta).$$

The component velocities immediately after the impact will be

$$ev_1\cos\alpha_1 \text{ horizontally},\quad ev_1(K\sin\alpha_1 - C\cos\alpha_1) \text{ vertically},$$

where, if e is the coefficient of restitution and μ is the coefficient of friction during the impact, $K = (5/7e)$ and $C = 0$ if $\mu > [2\tan\alpha_1/7(1+e)]$ or $K = 1/e$, $C = [\mu(1+e)/e]$ if $\mu < [2\tan\alpha_1/7(1+e)]$. [See Routh, *Elementary Rigid Dynamics*, Volume I, § 197. The ball is supposed homogeneous.]

Consequently, t seconds after the impact, the ball will have moved from the point of impact x horizontally and y vertically, where (supposing downward motion not to have begun) at the time of impact

$$x = ev_1\cos\alpha_1\cdot t,\quad y = ev_1(K\sin\alpha_1 - C\cos\alpha_1)t - \tfrac{1}{2}gt^2,$$

and it is required that when $x = a$, and therefore $t = (a/ev_1\cos\alpha_1)$,

$$y \text{ shall be } = -y_1 = -a\tan\beta + \frac{a^2g}{2v^2}\sec^2\beta,$$

i. e.,

$$a(K\tan\alpha_1 - C) - \frac{a^2g}{2e^2v_1^2}\sec^2\alpha + a\tan\beta - \frac{a^2g}{2v^2}\sec^2\beta = 0.$$

If we substitute in this equation the values of v_1^2 and $\tan\alpha_1$ given above, we shall have, after simplifying:

$$2e^2[(1+k)\tan\beta - C]v^6 - ag[(1+e^2+2Ke^2)\sec^2\beta + 4(1+K)e^2\tan^2\beta - 4e^2C\tan\beta]v^4$$
$$+ 2a^2g^2\sec^2\beta[(1+2e^2+3Ke^2)\tan\beta - e^2C]v^2 - a^3g^3(1+e^2+2Ke^2)\sec^4\beta = 0.$$

If we let

$$\frac{1+e^2+2Ke^2}{(1+K)\tan\beta - C} = 4e^2r \quad\text{and}\quad \frac{v^2}{ag} = x,$$

we shall have

$$x^3 - 2(r\sec^2\beta + \tan\beta)x^2 + \sec^2\beta(4r\tan\beta + 1)x - 2r\sec^4\beta \equiv$$
$$(x - 2r\sec^2\beta)(x^2 - 2\tan\beta x + \sec^2\beta) = 0.$$

The only real root is

$$x = \frac{v^2}{ag} = 2r \sec^2 \beta.$$

In case the ball at the time of impact has passed the highest point of its path ($v^2 \sin 2\beta < 2ag$), the problem is clearly impossible; this may be made to appear by putting $(-C)$ in place of (C) in the foregoing discussion.

There are two cases when $v^2 \sin 2\beta > 2ag$:

I. If

$$\frac{7}{2}\mu(1+e) > \tan \alpha_1, \quad v^2 = ag \csc 2\beta \left[\frac{7 + 10e + 7e^2}{e(7e+5)}\right].$$

II. If

$$\frac{7}{2}\mu(1+e) < \tan \alpha_1, \quad v^2 = ag \sec^2 \beta \frac{1+e}{2e(\tan \beta - \mu)}.$$

To these may be added:

III. If $\mu = 0$,

$$v^2 = ag \frac{1+e}{e} \operatorname{cosec} 2\beta.$$

By means of the relation

$$\tan \alpha_1 = \tan \beta - \frac{ag}{v^2} \sec^2 \beta = \tan \beta \left(1 - \frac{2ag}{v^2 \sin 2\beta}\right),$$

combined with the values of v^2 in the two cases, we find that Case I (rolling impact) or Case II (rolling and sliding at impact) occurs, according as $\mu \cot \beta$ is greater than or less than $[2(1-e)/7 + 10e + 7e^2]$.

If the ball is not homogeneous, the criterion for Case I and Case II is

$$\mu > \text{or} < \frac{k^2}{a^2 + k^2} \cdot \frac{\tan \alpha_1}{1+e},$$

and K becomes $a^2/e(a^2 + k^2)$, where k is the radius of gyration for a diameter. The discussion is otherwise unchanged, so that in this more general case

$$v^2 = ag \operatorname{cosec} 2\beta \cdot \frac{2a^2e + (a^2+k^2)(1+e^2)}{a^2e + (a^2+k^2)e^2}$$

or

$$v^2 = ag \sec^2 \beta \cdot \frac{1+e}{2e(\tan \beta - \mu)},$$

according as $\mu \cot \beta$ is greater or less than

$$\frac{k^2(1-e)}{2a^2e + (a^2+k^2)(1+e^2)}.$$

Also solved by J. A. Caparo, A. M. Harding, and Joseph B. Reynolds.

Note.—No solution of 300 has been received. H. S. Uhler should have received credit for solving 297 and 298. Editors.

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

NEW QUESTIONS.

30. A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high-school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high-school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

Note.—In transmitting this question the proposer writes, "The mathematical courses of our colleges seem to be designed chiefly for two classes of students, those expecting to pursue the